That is not my dog: Why doesn't the log dividend-price ratio seem to predict future log returns or log dividend growths? [1]

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Abstract: According to the "accounting identity" of Campbell and Shiller, the changes in log dividend-price ratio must predict either future returns, future dividend growth, or both. It is a well-known puzzle that neither seems to be predictable. We look at the analysis of this problem step-by-step starting with the single-period decomposition of returns, its log-linearization, accumulation over time, stationarity of log dividend-price that implies we can drop the final term in the limit, dropping the final term in the sample we have, and then econometric estimation. Although Campbell and Shiller's test of stationarity of the log dividend-price ratio does not make sense (because the log dividend-price ratio is nonstationary in both the null and the alternative of their test) and our corrected test does not reject non-stationarity in the sample we have now, the non-stationarity is not a big problem to date and the log-linearization performs adequately. Instead, the main problem is in the power of the empirical tests that have been used, which use one or a few lags. Using a test with a lag structure motivated by the theory, we find that the log dividend-price ratio significantly predicts dividend growth but does not significantly predict returns, based on a test that does the appropriate correction for overlapping data and spurious regression bias. What is happening is that there is small predictability of dividend growth spreading over many periods, and when we look at predictability over one or a few periods the predictability is hidden in the noise.

Key words: return predictability, dividend-price ratio, stationarity test. [JEL G12 G17]

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Clouseau: Does your dog bite?

Innkeeper: No.

Clouseau: Nice doggy.

(Clouseau tries to pet the dog on the floor and is bitten)

Clouseau (angry): I thought you said your dog did not bite.

Innkeeper: That is not my dog. [2]

Many times, we get into trouble because we ask the wrong question. In this paper, we look at the paradox of the failure of the predictability by the log dividend-price ratio (hereafter LDPR) derived by Campbell and Shiller (1988) on both future log returns and future log dividend growth. Because we do not know which question to ask, we go step-by-step through the entire argument to uncover where the problem is. In particular, the "accounting identity" of Campbell and Shiller asserts that current LDPR is approximately equal to a constant plus the sum of present values of future log returns minus the sum of present values of future log dividend growths, which implies that current LDPR should be able to predict future returns, or dividend growth rates, or both. The Campbell-Shiller approximation and its implications are applied widely in literature, particularly in stock return return predictability tests. The literature has a seeming contradiction: the theory says LDPR must predict either future returns or future dividend growth, but empirically it seems to predict neither. We go step-by-step through all the steps of the argument: theory, approximation, and empirical testing. Based on the data available, the largest problem is the lack of power of existing tests (which look at relatively near-date predictability) and we find that dividend growth is significantly predictable if we look at a test that mirrors more accurately the theory. We also find an error in the Campbell-Shiller test of stationarity of the log dividend-price ratio, and if we correct it we cannot reject nonstationarity over the whole period. While this is a conceptual problem and possibly an issue in the future, the Campbell-Shiller approximation works pretty well (looking out 30 years) in the current sample.

We reconcile the inconsistency between theoretical prediction and empirical findings by evaluating the so-called the Campbell-Shiller "accounting identity" step by step from the definition of returns through the approximation and its quality in one period and many periods, elimination of the

[2] Metro-Goldwyn-Myers, 1976. The Pink Panther Strikes Again.Amjo Productions, Video, 1976. http://www.dailymotion.com/video/x1w0z3c_the-pink-panther-strikes-again-1976-full-movie_shortfilms

final term, and estimation that is consistent with the theory. Using the annual dividend payment and price of $S\&P500$ index from 1871 to 2015, we find that current LDPR can be effectively and linearly approximated by the sum of weighted log returns and the sum of weighted dividend growth rates over long-run future periods plus the LDPR in the final period. Based on a regression of LDPR on the sums and the final log dividend-price ratio, all coefficients on independent variables are close to one and the R^2 is close to 100%. The Campbell-Shiller approximation implies that the current LDPR is able to predict either future returns or dividend growth or both; we find that only dividend growth is predictable. Moreover, dividend predictability spreads over many periods, which cannot be captured by conventional simple regression or vector-autoregressive (VAR) estimation.

Furthermore, we follow a conventional stationarity test in which the alternative hypothesis is that the underlying series is stationary, rather than the incorrect Campbell-Shiller stationarity test. In the Campbell-Shiller test, the LDPR series is nonstationary both under the null (unit root plus a trend) and under the alternative (stationary plus a trend). Instead, we use a Dickey-Fuller test without the trend, so that we have nonstationarity under the null but not under the alternative and we cannot reject nonstationarity (which may mean the long-term mean does not exist) when we use the entire sample. In principle, this is a big problem for the Campbell-Shiller approximation, which is based on an expansion around the long-term mean, but the approximation still works pretty well looking 30 years out in our sample. Furthermore, the approximation is worse but not so bad if we drop the final term after 30 years.

The final step of our analysis is to look at the empirical prediction of the model. We do this by testing an equation that looks like the equation in the model instead of using one or a few lags as is common in the literature. We do this using corrections for the correlation in error terms and spurious regression bias. We find that future log dividend growth is significantly predictable, but future returns are not. As noted by Cochrane (2008), dividends are smooth. He concludes that log dividend growth is not predictable (implying under the model restriction that returns are predictable). However, it is more accurate to assert that predictability of log dividend growth is spread over many maturities and nearby dividends are not very predictable because dividends are smooth. The limitation of using small lags to find predictability of dividend growth seems to be a solution of the puzzle of why the theory (based on an accounting identity and an approximation that is not so bad in the current sample) has not been verified.

The rest of this paper is organized as the follows. We review the approximation leading to the accounting identity Section 1 and test the quality of the approximation in Section 2. We propose a model-implied novel approach to test the predictability of return and dividend growth in Section 3. We conduct robustness analyses in Section 4 and further analyses to understand the predictability of dividend growth in Section 5. We analyze whether the failure of predictability of stock returns is caused by noise in itself or noise introduced by modeling procedure in Section 6. We conclude the paper in Section 7.

1 Dividend-Price Decomposition

We begin by specifying the standard definition relating return, future price, and dividend payment. Define gross investment return over one period as:

$$(1) \quad 1+R_{t+1} = \frac{P_{t+1}+D_{t+1}}{P_t} = \frac{P_{t+1}}{P_t}\left(1+\frac{D_{t+1}}{P_{t+1}}\right),$$

where P_t and P_{t+1} denote respectively the stock prices at the begin and the end of the period, and R_{t+1} and D_{t+1} denote respectively the net return over the period and dividend payment at the end of the period, abstracting from splits and distributions other than dividends.[3] This may seem like a strange way to write the return, since we normally would look at gross capital gains P_{t+1}/P_t and dividend yield D_t/P_t, with information known at the beginning of the period in the denominator, but we are simply manipulating accounting identities, and taking P_t in both denominators would give us a telescoping series in which the final term does not vanish. Taking logs on both sides, equation (1) becomes:

$$(2) \quad \log(1+R_{t+1}) = \log\left(\frac{P_{t+1}}{P_t}\right) + \log(1+\delta_{t+1}),$$

[3] For empirical work, we might treat all dividend payments during the period as coming at the end of the period, or we might try to do some more correct return calculation taking into account the timing of the dividends and the returns in sub-periods. In practice it probably doesn't matter much.

where $\delta_{t+1} \equiv \log(D_{t+1}/P_{t+1})$. We will approximate equation (2) by a first-order Taylor series expansion around $\delta_{t+1} = \delta$. Traditionally, δ is taken to be the long-term mean of the LDPR $\log(D_t/P_t)$, but we will take a broader view and think about δ being some reasonable value, since we will present some evidence that the long-term average may not exist. Letting $\rho \equiv 1/(1+\exp(\delta))$, then

$$\begin{aligned}\left.\frac{d\log(1+\exp(\delta_{t+1}))}{d\delta_{t+1}}\right|_{\delta_{t+1}=\delta} &= \left.\frac{\exp(\delta_{t+1})}{1+\exp(\delta_{t+1})}\right|_{\delta_{t+1}=\delta} \\ &= 1-\rho.\end{aligned}$$

Therefore, letting $\kappa \equiv \log(1-\exp(\delta)) - (1-\rho)\delta$, the Taylor approximation is:

$$\begin{aligned}(3)\quad \log(1+R_{t+1}) &= \log\left(\frac{P_{t+1}}{P_t}\right) + \log(1+\exp(\delta_{t+1})) \\ &\approx \log(\frac{P_{t+1}}{P_t}) + \log(1+\exp(\delta)) + (1-\rho)(\delta_{t+1}-\delta) \\ &= \log(P_{t+1}) - \log(P_t) + \log(1-\exp(\delta)) \\ &\quad +(1-\rho)(\log(D_{t+1}) - \log(P_{t+1})) - (1-\rho)\delta \\ &= \kappa + \rho\log(P_{t+1}) + (1-\rho)\log(D_{t+1}) - \log(P_t).\end{aligned}$$

We follow Campbell and Shiller (1988) in being informal about the sense of the approximation; we will take an empirical approach to determine how well the approximation works. We can rewrite equation (3) as

$$(4)\quad \log\left(\frac{D_t}{P_t}\right) \approx -\kappa + \log(1+R_{t+1}) + \rho\log\left(\frac{D_{t+1}}{P_{t+1}}\right) - \Delta\log(D_{t+1}).$$

Substituting the same for $t+1$, $t+2$, and so forth for the $\log(D_{t+1}/P_{t+1})$ on the right-hand side telescopes to imply:

$$(5)\quad \log\left(\frac{D_t}{P_t}\right) \approx -\frac{\kappa}{1-\rho}(1-\rho^{T-t}) + \sum_{s=t+1}^{T}\rho^{s-t}(\log((1+R_s) - \Delta\log(D_s)) + \rho^{T-t}\log\left(\frac{D_T}{P_T}\right).$$

This is the essential relationship that we will work with. Since $\rho < 1$ it is at least plausible to argue (as do Campbell and Shiller) that the final term should vanish as T increases, and we have the asymptotic expression

$$(6) \quad \log\left(\frac{D_t}{P_t}\right) \approx -\frac{\kappa}{1-\rho} + \sum_{s=t+1}^{\infty} \rho^{s-t}(\log((1+R_s) - \Delta\log(D_s)),$$

often referred to in the literature as *the accounting identity*. This says that, subject to the quality of the approximation, today's log dividend-price ratio $\log(D_t/P_t)$ is *identically equal to* to a linear combination of future log returns $\log(1+R_s)$ and future changes in log dividends $\Delta\log(D_s)$. This implies that the log dividend-price ratio must predict one or both of these. The puzzle in the literature is that the log dividend-price ratio seems to predict neither future log returns nor future log dividend changes.

2 Approximation Test

Before testing whether stock returns are predictable, we test the quality of the LDPR approximation in equation (5) with the annual prices and aggregate dividend payments of the S&P 500 index firms over 1871 to 2015.[4] We focus on annual data because monthly dividend payments are linearly interpolated from annual and quarterly dividend payments, and we do not want to deal with the approximation error this might entail. Over the same period, the average gross return is 10.56% with a standard deviation of 18.17% , and the average annual log gross return on the S&P 500 index is 8.61% with a standard deviation of 17.33%; the average dividend-price ratio is 4.47% with a standard deviation of 1.52% while the average log dividend price ratio is -3.18 with a standard deviation of 0.40; and the average annual log dividend growth rate is 4.37% with a standard deviation of 12.16%.

Campbell and Shiller (1988) suggest a vector autoregression (VAR) approach to test the equation (5) without the final term instead of the conventional predictive regression and find that the LDPR series is persistent and is able to predict both future stock returns and future dividend growth but the associated R^2s in their tests are small. We replicate and confirm their results. Although Campbell and Shiller claim that the VAR procedure does a better job than single linear regressions to "detect long-term deviations of stock prices from the 'fundamental value' ", the VAR approach suffers several

[4]The data is collected from Robert Shiller's website at http://www.econ.yale.edu/~shiller/data.htm. We thank Shiller for making it available online.

shortcomings. First, a VAR procedure with limited number of lags does not sufficiently capture the long-term relationship among current dividend-price ratio, future returns and future dividend growth rates. Cochrane (2011) shows that VAR estimates can be biased and significantly different from that in the true linear regressions. Second, the calculations of a VAR procedure will be much more complicated for more than two variables with even very limited lags. Finally, this procedure ignores the final term in the equation. To conclude, the analysis in Campbell and Shiller (1988) does not sufficiently tell whether (5) holds empirically.

An improved approach to avoid such shortcomings is to conduct a true linear regression of log dividend-price ratio on $2(T-t)$ terms of discounted log return and dividend growth plus one final term. This procedure, however, is burdensome and may not be implementable when $(T-t)$ is large and the sample period is not sufficiently long. In this study, we propose a parsimonious regression of current LDPR on the sum of weighted future returns, the sum of weighted dividend growth rates and the log dividend-price ratio in the last period specified as the following:

(7)

$$\log\left(\frac{D_t}{P_t}\right)=\alpha+\beta_1\left(\sum_{s=t+1}^{T}\rho^{s-t}(\log(1+R_s))\right)+\beta_2\left(\sum_{s=t+1}^{T}\rho^{s-t}\Delta\log(D_s)\right)+\beta_3\left(\rho^{T-t}\log(\frac{D_T}{P_T})\right)+\varepsilon_t.$$

By construction, this regression overcomes the shortcomings in both conventional linear and VAR estimations and is more parsimonious. If the approximation of equation (5) is effective, we should expect that all estimated coefficients (βs) in equation (7) are close to one (or negative one) and the corresponding R^2 is around 100%.

We take $(T-t)$ to be 30 years, which is reasonably long and gives us 115 overlapping observations (years) for analysis. The results are reported in the first column in Table 1 and suggest that the LDPR approximation in equation (5) is effective. The coefficient on the sum of discounted return is positive and close to one, and the coefficient on the sum of discounted dividend growth is negative and approximately equal to -1. All coefficients are statistically significant at 1% level and the corresponding R^2 is as high as 99%. This regression uses Newey-West standard errors to adjust for serial correlation (including that due to overlapping observations) and heteroscedasticity. The high R^2 suggests that current LDPR predicts at least one regressor but not which one(s). Note that we have

Table 1: Approximation Test

This table reports the empirical results of whether the approximation of log dividend-price ratio in equation (5) is effective. The results are based on the annual prices of and dividend payments on the S&P 500 index from 1871 to 2015. The $(T-t)$ is set to be 30 years. The associated Newey-West standard error with four lags are in parentheses. *** denotes statistical significance at the 1% level.

	Model 1	Model 2
α	-2.96^{***}	-3.64^{***}
	(0.05)	(0.08)
β_1	1.01^{***}	0.93^{***}
	(0.02)	(0.08)
β_2	-1.04^{***}	-1.14^{***}
	(0.02)	(0.09)
β_3	1.19^{***}	
	(0.05)	
N	115	115
R^2 (%)	98.94	82.57

not adjusted for spurious regression bias (caused by low frequency series on both sides). For now, it suffices to note that the fit is very good. We will correct for spurious regression bias when we conduct predictive regressions.

To address the concern raised by Kleidon (1986), Marsh and Merton (1986), and Merton (1987), we further test whether the final term in equation (5) is small by repeating the analysis on equation (7) after dropping the term. The results are reported in the second column of Table 1 and suggest that the log dividend-price ratio in the final period (30 years from now) is not trivial but also not so large. The coefficients on the sum of discounted returns and dividend growth are still significant but slightly deviate from 1 (or -1), and the R^2 drops significantly by 16%, from 99% to 83%, evidence that equation (7) is a good specification for empirical estimating of log dividend-price ratio.

3 Predictability Test

Equation (5) suggests a predictive relationship of current LDPR on cumulative future log returns or cumulative log dividend growth rates (for example, Campbell and Shiller, 1988; Cochrane, 2011). Furthermore, equation (5) suggests that the true predictive tests should be conducted by reversing the dependent and independent variables in equation (7) as:[5]

$$(8)\quad \sum_{s=t+1}^{T} \rho^{s-t}(\log(1+R_s)) = \alpha + \beta_1 \log\left(\frac{D_t}{P_t}\right) + \mu_T.$$

A significant β_1 and a reasonable level of R^2 would suggest a predictable relationship between the LDPR and the cumulative future returns. This specification applies to the predicability of future log dividend growth rates. The use of cumulative present values of the predicted variable in future periods has advantages against conventional predictive specification, in which one-period leading predicted variable is mostly used, in that it can capture the total predictability of future return or dividend growth. In other words, equation (8) captures both short-run and long-run return predictabilities (if any).

Equation (5) implies that the sum of all coefficients on the log dividend-price ratio (β_1s) across all three predictive tests (i.e. predictability tests of $\sum_{s=t+1}^{T} \rho^{s-t}(\log(1+R_s))$, $\sum_{s=t+1}^{T} \rho^{s-t}\Delta\log(D_s)$ and $\rho^{T-t}\log(\frac{D_T}{P_T})$) should be one if our predictive specification is appropriate.[6] However, it is not possible to explore this relationship with three conventional predictive regressions. Granger and Newbold (1974), Stambaugh (1999), and Ferson, Sarkissian, and Simin (2003) show that linear regressions with lagged stochastic regressors with finite sample suffer spurious regression bias (SRB). We follow the approach proposed by Stambaugh (1999) to estimate the spurious regression bias, and calculate the SRB-adjusted coefficients on the predictor (i.e. the LDPR). The empirical results are reported in Table 2.

The sum of all LDPR coefficients from the three predictability tests of $\sum_{s=t+1}^{T} \rho^{s-t}(\log(1+R_s))$, $\sum_{s=t+1}^{T} \rho^{s-t}\Delta\log(D_s)$ and $\rho^{T-t}\log(D_T/P_T)$ is close to one $(0.19 - (-0.57) + 0.17 = 0.93)$ and

[5]In the presidential speech for the *AFA* 2011 annual meeting, Cochrane also recommends this return predictability test specification while he does not conduct this type of analysis.

[6]This relationship is also perceived by Cochrane (2011).

Table 2: Predictability Test

This table reports the empirical results of whether current log-dividend-price ratio is able to predict the sum of discounted future returns, the sum of discounted future dividend growths, or discounted log dividend-price 30 years from now. The results are based on the annual data of the S&P 500 index from 1871 to 2015. The spurious regression bias (SRB) is estimated following Stambaugh (1999). The associated Newey-West standard error with four lags are in parentheses. *** and ** denote statistical significance at the 1% and 5% levels, respectively.

Predicted variable	α	β_1	SRB-adjusted β_1	R^2 (%)
$\sum_{s=t+1}^{T}\rho^{s-t}(\log(1+R_s))$	1.88**	0.19	0.17	1.30
	(0.72)	(0.23)		
$\sum_{s=t+1}^{T}\rho^{s-t}\Delta\log(D_s)$	−1.20*	−0.57***	−0.61	15.54
	(0.62)	(0.20)		
$\rho^{T-t}\log(\frac{D_T}{P_T})$	−0.19	0.17***	0.17	18.03
	(0.18)	(0.06)		

the corresponding SRB-adjusted predictor coefficient sum is similar and even closer to one (0.95), evidence that our predicative specifications are theoretically appropriate and that spurious regression bias is not severe in our analysis. More interestingly, Table 2 shows that the LDPR (predictor) coefficient (row 1) in the predictability test of cumulative future returns is insignificant while it is significant in the predictability tests of cumulative future dividend growth (row 2) and the discounted final-period log dividend-price ratio (row 3). The size of the coefficient on LDPR in the predictability test of future dividend growth rates dominates the other two counterparts and contributes 61% (0.57/0.93) to the sum of all three predictor coefficients. The predictor coefficient in the predictability test of future returns contributes 20% to the sum of all three predictor coefficients. Similar patterns hold in the R^2s for both coefficients. The predictor coefficient in the predictability test of the final-period dividend-price ratio is small but the corresponding R^2 is the highest (18%). The results provide significant evidence to reject the hypothesis that future dividend growth rates are not predictable but not the hypothesis that future return rates are not predictable. The main reason, as will be soon shown below, is that stock return generating process is too noisy to be predictable, consistent with Shiller (1981), and Poterba and Summers (1988) that stock prices are too volatile to be explained by fundamentals. The unpredictability of stock returns supplements the argument by Lanne (2002), Valkanov (2003), and Boudoukh, Richardson and Whitelaw (2008) that conventional analysis of long-term predictability of stock returns is spurious. Moreover, the results from Table 2 suggest that the relationship among the three predictor coefficients (Cochrane, 2011) is not informative about stock return predictability.

In the meantime, it is worth further exploring why the coefficient on stock returns in Table 1 is consistently close to one and statistically significant but the predictor coefficient (on LDPR) in stock return predictability test in Table 2 is small and insignificant. Our explanation is that the coefficient of one in the first case is caused by high collinearity between stock return and dividend growth rather than information innovation in stock return generating process. To illustrate our argument, let start with equation (5) as $log(D_t/P_t) \approx \alpha + \beta_1(\sum_{s=t+1}^{T}\rho^{s-t}(\log(1+R_s))) + \beta_2(\sum_{s=t+1}^{T}\rho^{s-t}\Delta\log(D_s)) + \varepsilon_t$. Assume that the first term only contains noise (denoted as Z_t) and the second term contains both information and noise as:

(9) $$\sum_{s=t+1}^{T}\rho^{s-t}(\log(1+R_s)) \approx Z_t.$$

(10) $$\sum_{s=t+1}^{T}\rho^{s-t}\Delta\log(D_s) \approx \log(\frac{D_t}{P_t}) + Z_t.$$

Then the covariance matrix between LDPR and sum of discounted future dividend growth rates becomes:

(11) $$\text{var}\left(\log\left(\frac{D_t}{P_t}\right), \sum_{s=t+1}^{T}\rho^{s-t}\Delta\log(D_s)\right) = \begin{pmatrix} \sigma_\delta^2 & -\sigma_\delta^2 \\ \sigma_\delta^2 & \sigma_\delta^2 + \sigma_Z^2 \end{pmatrix}.$$

When we run current LDPR on the sum of discounted future dividend growth rates as $\log(\frac{D_t}{P_t}) = \alpha + \beta(\sum_{s=t+1}^{T}\rho^{s-t}\Delta\log(D_s)) + \mu_t$, the coefficient on the independent variable is given as: $\beta = \frac{-\sigma_\delta^2}{\sigma_\delta^2+\sigma_Z^2}$. When the noise σ_Z^2 in stock return is large, then the coefficient will be downward biased. Moreover, the correlation between the sum of discounted future returns and the sum of discounted future dividend growth also becomes large.

$$(12)\quad \text{var}\left(\sum_{s=t+1}^{T}\rho^{s-t}(\log(1+R_s)),\sum_{s=t+1}^{T}\rho^{s-t}\Delta\log(D_s)\right)=\begin{pmatrix}\sigma_\delta^2 & \sigma_Z^2\\ \sigma_Z^2 & \sigma_\delta^2+\sigma_Z^2\end{pmatrix}.$$

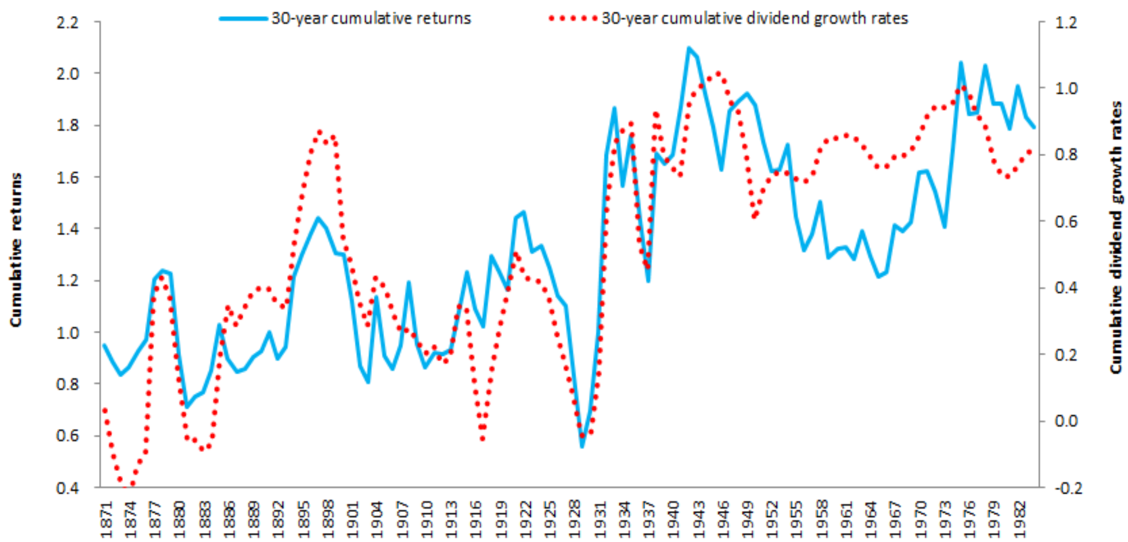
$$(13)\quad \text{corr}\left(\sum_{s=t+1}^{T}\rho^{s-t}(\log(1+R_s)),\sum_{s=t+1}^{T}\rho^{s-t}\Delta\log(D_s)\right)=\frac{\sigma_Z^2}{\sigma_Z^2+\sigma_\delta^2}.$$

Equation (9) implies that when we conduct a stock return predictability test as $\log(\frac{D_t}{P_t})=\alpha+\beta(\sum_{s=t+1}^{T}\rho^{s-t}(\log(1+R_s)))+\mu_t$, the β coefficient should be close to zero and our untabulated empirical results confirm this. However, equation (13) suggests that when the noise is large, we may end up with a spurious coefficient significantly different from zero, which suggests that the approximation test of LDPR specified in equation (5) is different from the predictive tests specified in equation (8). Our explanation is consistent with the empirical data. Figure 1 shows that the evolutions of cumulative log returns and dividend growth rates are closely correlated. In fact, the correlation between the log return and the log dividend growth is 0.63, and the correlation between $(\sum_{s=t+1}^{T}\rho^{s-t}(\log(1+R_s)))$ and $(\sum_{s=t+1}^{T}\rho^{s-t}\Delta\log(D_s))$ is 0.84. Moreover, the standard deviations of the cumulative returns and log dividend growth rates are respectively 38.1% and 34.3% which implies that sum of variance of the two terms is as high as 37.2%, or 61% in terms of standard deviation, while the standard deviation of the log dividend-price ratio over the same period is 23.6%. This spurious relationship is also observed by Ferson, Sarkissian and Simin (2003) and Valkanov (2003) with simulated data.

4 The Long-Term Mean Log Dividend-Price Ratio

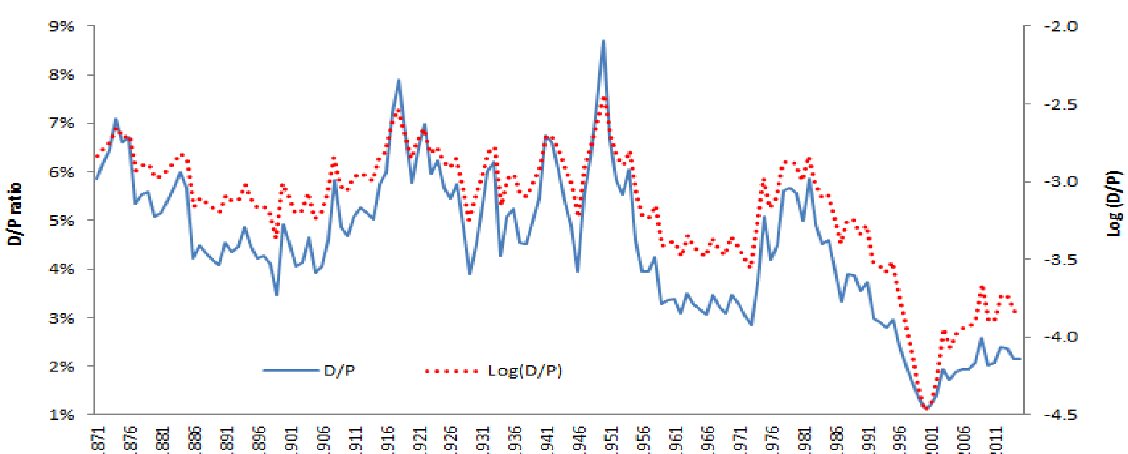
The empirical analyses in the previous section are based on expanding LDPR around its sample mean. The literature typically assumes that the long-term mean exists and that the expansion is around the long-term mean. Unfortunately, Campbell and Shiller's test of existence of the long-term mean does not make sense, because the long-term mean does not exist either under the null or under the alternative of their test because both include trends. In a corrected version of their test without a trend,

Figure 1: Time Series of Cumulative Discounted Returns and Dividend Growth Rates.



we cannot reject the null of nonstationarity under which the long-term mean LDPR does not exist. This certainly weakens the interpretation of the sample mean as the long-term mean, but it doesn't necessarily invalidate the analysis using the current data, as we will discuss in the next Section. The time series of dividend-price ratio of S&P 500 index (dot line) and the corresponding log ratio (solid line) are plotted in Figure 2. Both show a strong declining trend. Specifically, the dividend-price

Figure 2: Time Series of $\frac{D}{P}$ and $log(\frac{D}{P})$.



ratio over Campbell-Shiller period (1871-1986) is around 5%, and declines to around 2% over post

Campbell-Shiller period (1987-2015), and becomes lower than 2% over the last 15 years in the 21^{st} century. Figure 2 suggests that the long-term mean of dividend-price or log dividend-price may not exist. To formally test the existence of LDPR long-term mean, we follow the conventional stationarity test in which the null is that LDPR is a nonstationary process. Although Campbell and Shiller claim to conduct a stationarity test, both their null and alternative hypotheses contain trends and hence theirs is not a test of stationarity. Our stationarity test is specified as $\log(D_t/P_t) = \alpha + \beta \log(D_{t-1}/P_{t-1}) + \varepsilon_t$ and the trend assumption is removed. The results are reported in Table 3.

Table 3: Stationarity Tests

This table reports the empirical results of whether the annual series of log dividend-price ratio of the S&P 500 index is stationary over the whole sample period, the Campbell-Shiller period, and the post Campbell-Shiller period. The stationarity test is specified as $\log(D_t/P_t) = \alpha + \beta \log(D_{t-1}/P_{t-1}) + \varepsilon_t$. The whole sample period is from 1871 to 2015 and the Campbell-Shiller period is from 1871 to 1986.

$log(\frac{D}{P})_t$	$1871-2015$ (full sample)	$1871-1986$ Campbell-Shiller	$1987-2015$ post Campbell-Shiller
α	-0.16	-0.39	-0.34
	(0.06)	(0.09)	(0.15)
β	0.89	0.71	0.82
	(0.04)	(0.07)	(0.09)
Dicky-Fuller stat	-15.78	-32.95	-4.76
Dicky-Fuller critical	-16.30	-16.30	-14.60
N	139	111	26
$Adj-R^2$	77.14	49.55	72.37
Reject unit root	No	Yes	No

Consistent with Figure 2 that the dividend payment on the S&P 500 index declines over time, the LDPR time series is stationary over Campbell-Shiller period but non-stationary over the whole sample period and post-Campbell-Shiller period. The Dicky-Fuller statistic is -15.8 over the whole sample period, -33.0 over the Campbell-Shiller sample period, and -4.8 over the post Campbell-Shiller period, and the corresponding critical values at 5% level are respectively -16.3, -16.3 and -14.6. In short, we fail to reject the hypothesis that LDPR is a nonstationary series (without a long term mean). If LDPR does not have a long-term mean, the sample mean cannot be an estimate of the long-term mean (which does not exist). Practically speaking the failure to reject nonstationarity says that we don't know whether the long-term mean exists – perhaps it exists but the test does not have enough power or is misspecified – but even if it does exists, lack of power in the test suggests we do not have a long enough data series to get a good estimate. It will certainly be a problem over time if the long-term mean does not exist and the LDPR gets more and more dispersion that will make the Taylor approximation worse and worse. However, a Taylor expansion around the sample mean may still be useful with the sample we have, and that is what we test next. In fact, we will find that our main results are not sensitive to the choice of δ in a reasonable range. This also says our results are not being caused by a look-ahead bias due to constructing the covariates using the sample mean LDPR for the whole period.

4.1 Alternative Expanding Points: Approximation

Figure 2 shows that the sample mean of dividend-price ratio is smaller than but close to 5% over the whole sample period and decreases to lower than 2% over years in the 21^{st} century. We first test whether the LDPR approximation in equation (5) is effective when it is expanded around alternative points. We consider two expanding points to take into account the declining trend in LDPR: 3% and 2% and two cases of approximation: with and without the final-period log dividend-price ratio. The empirical test is specified in the same way as equation (7) and results are reported in Table 4.

When LDPR is expanded around 3% (Panel A), the coefficients on the sums of discounted future returns and dividend growth rates are almost unchanged relative to that in Table 1 and close to $1(-1)$ and the R^2 is slightly reduced by 0.5%. The R^2 is reduced by about 30% when the final-period log dividend-price ratio in equation (7) is dropped from the LDPR approximation test. These findings suggest that the approximation in equation (5) is effective when firm's payout is around a reasonable level, such as 5%, and that the log dividend-price ratio in the final period should not be dropped. When LDPR is expanded around 2%, which is close to the payout ratio over years in the 21^{st} century, Panel B in Table 4 shows that the precision of the LDPR approximation is slightly impacted but still reasonably good. The coefficients are close to 1 (-1) and the R^2 is around 94%. In short, Table 4 suggests that expanding LDPR around its sample mean is meaningful and effective with current sample even though its long-term mean does not exist.

Table 4: Approximation Tests: Alternative Expanding Points

This table reports the linear regression results of the Taylor expansion of log dividend-price ratio around alternative points. The regression is specified as: $\log(D_t/P_t) = \alpha + \beta_1(\sum_{s=t+1}^{T} \rho^{s-t}(\log(1+R_s))) + \beta_2(\sum_{s=t+1}^{T} \rho^{s-t} \Delta \log(D_s)) + \beta_3(\rho^{T-t} \log(D_T/P_T)) + \varepsilon_t$. The results are based on the annual data of the S&P 500 index from 1871 to 2015. The $(T-t)$ is set to be 30 years. The associated Newey-West standard error with four lags are in parentheses. *** denotes statistical significance at the 1% level.

	Expanding point: 0.03($\rho \approx 0.97$)		Expanding point: 0.02 ($\rho \approx 0.98$)	
	Model 1	Model 2	Model 1	Model 2
α	-2.83^{***}	-3.67^{***}	-2.71^{***}	-3.60^{***}
	(0.03)	(0.09)	(0.08)	(0.10)
β_1	0.99^{***}	0.77^{***}	0.94^{***}	0.63^{***}
	(0.02)	(0.07)	(0.04)	(0.07)
β_2	-0.96^{***}	-0.97^{***}	-0.88^{***}	-0.82^{***}
	(0.02)	(0.08)	(0.04)	(0.07)
β_3	0.94^{***}		0.82^{***}	
	(0.03)		(0.05)	
R^2 (%)	98.43	69.03	93.95	56.45

4.2 Alternative Expanding Points: Predictability

In this subsection, we test whether the findings on the predictability tests of future returns and dividend growth rates in Section (3) hold for different discounting rates (corresponding to different expanding points). The empirical results are reported in Table 5 and support that the predictability of future dividend growth rates and the unpredictability of future stock returns are not impacted by expanding points.

When the expanding point is set to be 0.03 (Panel A) the coefficient on current LDPR is small and statistically insignificant in the predictability test of future returns, and large and significant in the predictability test of future dividend growth rates. Moreover, the coefficient on current LDPR becomes negative and insignificant in the stock return predictability test when LDPR is expand around the point of 0.02 (Panel B), which is close to the level in 2100s, suggesting that stock return is not predictable at all when LDPR becomes small. The magnitude and significance of the coefficient on LDPR in the predictability test of dividend growth rates are unchanged, suggesting that dividend growth rate is still predictable even when LDPR is small.

5 The Term Structure of Dividend Growth Predictability

The analyses so far suggest that the lack of predictability on future returns is due to noise in returns and the predictability on future dividend growth is due to that current LDPR contains future dividend policy information. Moreover, literature shows that dividend growth follows a persistent pattern, which implies that low current LDPR reflects predictable upward adjustments over multiple future periods and vice versa. We test whether current LPDR can predict future dividend growth across multiple periods by splitting the sum of discounted dividend growth over future 30 years (see Table 2) into sums over respectively three non-overlapping 10 years. The results are reported in Panel A in Table 6 and suggest that the predicability of dividend growth spreads in next twenty years. The coefficients on LDPR are statistically significant in predicting both the subsequent first and second ten years (the 1^{st} and 2^{nd} rows). The coefficient on LDPR in predicting subsequent third ten years (the 3^{rd} row) is close to zero and insignificant.

Using a simple linear regression approach, Chen (2009) shows that dividend growth rate is not

Table 5: Predictability Tests: Alternative Expanding Points

This table reports the empirical results of whether current log dividend-price ratio is able to predict sums of discounted future returns or discounted dividend growth rates, or the discounted final-period log dividend-price ratio. The results are based on the annual data of the S&P 500 index from 1871 to 2015. The spurious regression bias (SRB) is estimated following Stambaugh (1999). The associated Newey-West standard error with four lags are in parentheses. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

Predicted variable	α	β_1	SRB-adjusted β_1	R^2 (%)
Panel A: Expanding point: 0.03 ($\rho \approx 0.97$)				
$\sum_{s=t+1}^{T} \rho^{s-t}(\log(1+R_s))$	1.82^{**}	0.03	0.01	0.03
	(0.86)	(0.28)		
$\sum_{s=t+1}^{T} \rho^{s-t}\Delta\log(D_s)$	-1.37^{*}	-0.68^{***}	-0.72	15.90
	(0.76)	(0.24)		
$\rho^{T-t}\log(\frac{D_T}{P_T})$	-0.35	0.31^{***}	0.31	18.03
	(0.34)	(0.12)		
Panel B: Expanding point: 0.02 ($\rho \approx 0.98$)				
$\sum_{s=t+1}^{T} \rho^{s-t}(\log(1+R_s))$	1.74^{*}	-0.08	-0.07	0.14
	(0.96)	(0.31)		
$\sum_{s=t+1}^{T} \rho^{s-t}\Delta\log(D_s)$	-1.45^{*}	-0.75^{***}	-0.80	15.58
	(0.85)	(0.27)		
$\rho^{T-t}\log\left(\frac{D_T}{P_T}\right)$	-1.45^{***}	0.75^{***}	0.76	15.58
	(0.50)	(0.16)		

predictable after the Second World War ($WWII$). We test this finding with our approach and the results are reported in Panel B in Table 6. Although dividend growth rates are more predictable before $WWII$ and less predictable after, they are significantly predictable in both sub-periods. In other words, the predictability of dividend growth is not dominantly driven by a specific sub-period.

Table 6: Term Structure of Dividend Growth Predictability

Panel A reports the empirical analysis of the term structure of dividend growth predictability. We split the sum of discounted dividend growth rates over future 30 years into sums over three non-overlapping 10 years. Panel B reports the results of whether dividend growth predictability exists after the Second World War ($WWII$). The results are based on the annual data of the S&P 500 index from 1871 to 2015. SRB refers to spurious regression bias estimated following Stambaugh (1999). The associated Newey-West standard error with four lags are in parentheses. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

Predicted variable	α	β_1	SRB-adjusted β_1	R^2 (%)
Panel A: Term Structure of Dividend Growth Predictability				
$[t+1, t+10]$	-0.61	-0.29^{**}	-0.30	5.84
	(0.42)	(0.13)		
$[t+11, t+20]$	-0.84^{***}	-0.33^{***}	-0.34	24.09
	(0.31)	(0.10)		
$[t+21, t+30]$	0.24^{*}	0.04	0.03	1.25
	(0.13)	(0.04)		
Panel B: Predictability Across Subsample Periods				
Pre-WWII $[1871, 1945]$	-3.51^{***}	-1.26^{***}	-1.28	57.41
	(0.69)	(0.24)		
Post-WWII $[1946, 2015]$	0.46^{**}	-0.12^{**}	-0.13	12.23
	(0.19)	(0.06)		

6 Further Analysis

Previous analyses show that stock return predictability is buried by noise but the LDPR approximation around its sample mean is helpful and appropriate. The seemly controversial results suggest that the noise contained in stock return series is endogenous as illustrated in Section (3) rather than introduced by the LDPR approximation. It is interesting to understand whether and how much noise is introduced in the LDPR series generating process in equation (5). For this purpose, we start examining the noise in one-period approximation and then the cumulative noise in equation (5).

6.1 Noise in Single Period

Note that equation (3) is a Taylor expansion of log return around the LDPR long-term mean. The expansion around sample mean in our empirical analyses may introduce a significant amount of noise when the long-run mean does not exist. We examine whether exogenous noise is added to our empirical approximation with equation (3) and define the noise term as:

$$(14)\quad \xi_t = \kappa + log(\frac{D_t}{P_t}) - \rho \log(\frac{D_{t+1}}{P_{t+1}}) + \log(\frac{D_t}{D_{t-1}}) - log(1+R_t),$$

where ρ and κ are derived with the LDPR sample mean.

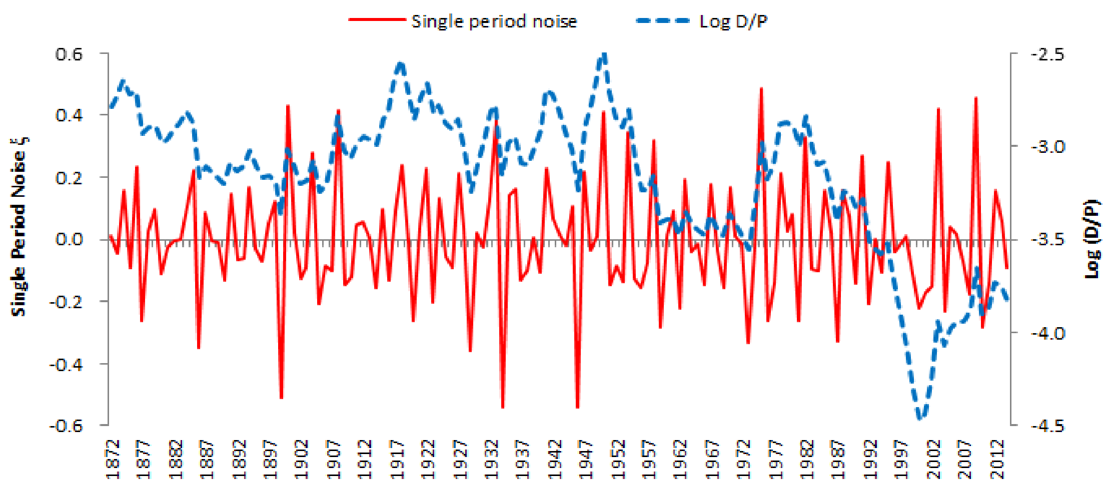
The time series of this single-period noise (ξ_t) is plotted in Figure 3.A and seems volatile. Although the mean of this noise term is close to zero (-0.37%) over the sample period from 1871 throughout 2015, its standard error is as high as 19.25%. An *ARMA* test suggests that the ξ_t series is an $AR(1)$ process. Although the magnitude of this noise is small relative to LDPR, which has a mean of -317.84% and a standard deviation of 40.44% over the same period, it is large relative to the log gross return, which has a mean of 8.61% and a standard deviation of 17.33%. Moreover, this noise will contaminate mainly stock return process because dividend growth process is more persistent. The results provide an explanation for findings in existing studies that short-term returns are not predictable by dividend-price ratio. In short, the approximation in equation (3) with the LDPR sample mean introduces a significant amount of noise in the process of returns and stock prices.

6.2 Noise in Multiple Periods

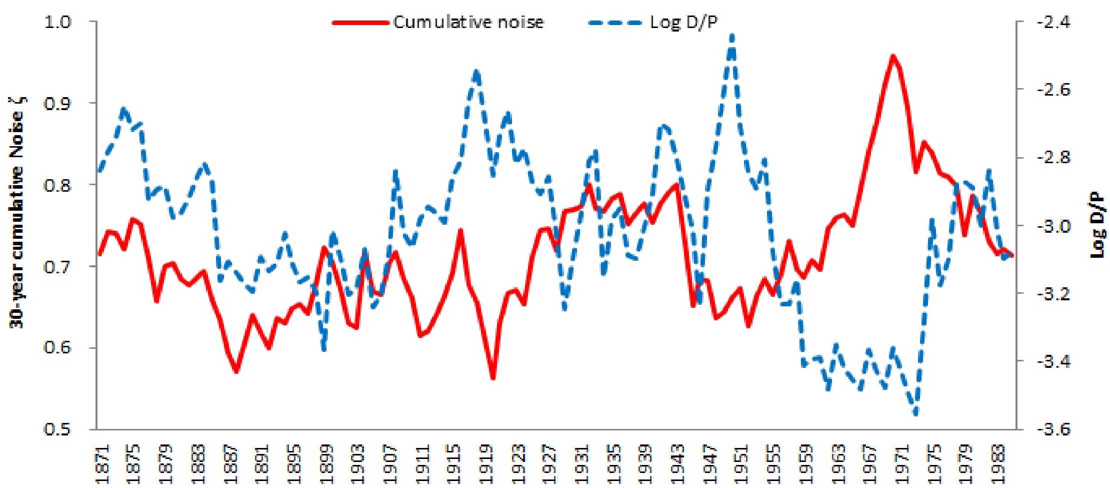
We further examine the cumulative approximation error in equation (5), which is defined as the following:

$$(15)\quad \zeta_t = -\frac{\kappa}{1-\rho}(1-\rho^{T-t}) + \sum_{s=t+1}^{T} \rho^{s-t}(\log((1+R_s) - \Delta \log(D_s)) + \rho^{T-t} \log(\frac{D_T}{P_T}) - log(\frac{D_t}{P_t}),$$

Figure 3: Time Series of Approximation Noise.



A Single-period Noise



B Multiple-period Noise

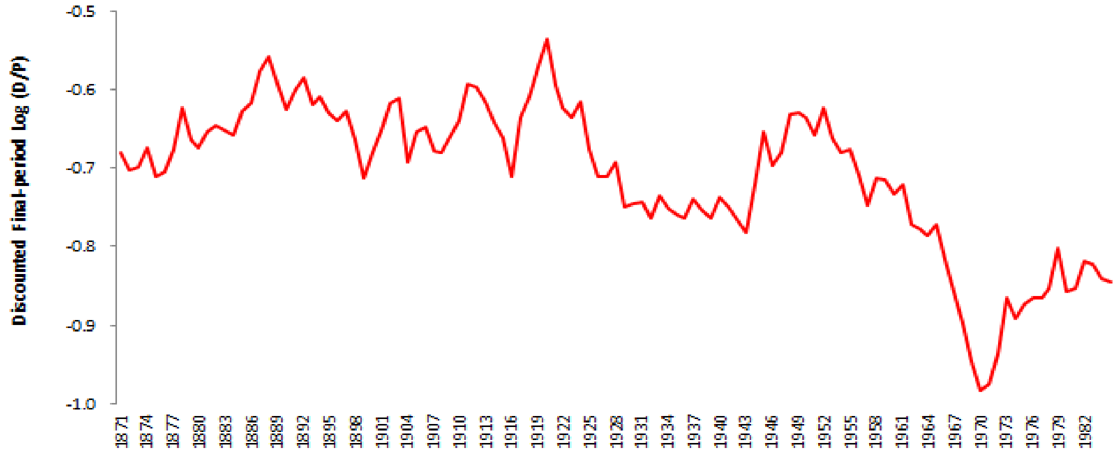
where ρ and κ are derived with the LDPR sample mean.

To be consistent with previous analyses, we take $(T-t)$ to be 30 and plot the evolution of ζ_t in Figure 3.B, which shows that the evolution of this noise term is smoother than the single-period noise and increases overtime. It has a sample mean of -1.5%, which is almost four times as large as that of one-period noise, and sample standard deviation of 3.5%, higher than that of the single-period noise either. An ARMA test shows that ζ_t is an $ARMA(1,1)$ process. In the meantime, the means of $log(\frac{D_t}{P_t})$,$\sum_{s=t+1}^{T}\rho^{s-t}(\log(1+R_s))$, $\sum_{s=t+1}^{T}\rho^{s}(\Delta\log(D_s))$ are respectively -3.02, 1.33 and 0.53, and their standard errors are 0.24, 0.38 and 0.34. Compared with the single-period noise, these numbers suggest that the approximation generated noise can be reduced by accumulating short-term returns. However, combining Figure 3 and Table 2 shows that our accumulating approach cannot reduce the endogenous noise in stock return generating process, suggesting that the lack of predictability of stock returns is caused by endogenous noise not by exogenously introduced noise. This is consistent with Shiller's (1981) finding that stock return is too volatile to be explained by fundamentals.

6.3 The Final Term

To simplify the LDPR approximation calculation, Campbell and Shiller (1988) suggest dropping the final term $\rho^{T-t}\log(D_T/P_T)$ in equation (5) and assign a large number for $(T-t)$. This simplification may make sense when $(T-t)$ is sufficiently large and ρ is not too large. In real life, however, it is interesting to see how big the $(T-t)$ needs to be. Base on our sample, the ρ is close to but slightly lower than 0.96. We take $(T-t)$ to be 30 years and ask whether this term is sufficiently small. The time series of this term with $(T-t)$= 30 is plotted in Figure 4 and suggest that this term is not too small in general but also not so large. In fact, the average of this term over the whole sample is as large as -0.71, larger (in absolute value) than the averages of log gross returns (0.09) or log dividend growth rates (0.04), and comparable to the averages of the 30-year discounted cumulative log gross returns (1.33) and log dividend growth rates (0.53).

Figure 4: Time Series of the Discounted Final-Period Log (D/P)



7 Conclusion

Whether stock returns are predictable is important and challenging to both academia and industry. Campbell and Shiller (1988) present an interesting argument, based on accounting definitions and some approximations, that the log dividend-price ratio must predict future returns, future log dividend growth, or both. However, neither prediction is significant economically or statistically, and this has been viewed as an important puzzle. We check each step of this argument, from the accounting definition through the approximation to the statistical tests. We find that although fixing an error in Campbell and Shiller's statistical analysis of a key assumption (stationarity of the log dividend-price ratio) might seem to invalidate the whole analysis, this is not important in the sample available to date. Rather, the source of the failure to find a significant relationship arises from a mismatch between the VARs in the traditional tests and the many terms in the theoretical expression. When we conduct a test closer to the theoretical expression, with appropriate correction for serial correlation due to overlapping data, the possible heteroscedasticity, and spurious regression bias, we find that future log dividend growth is significantly predictable but future returns are not, thus resolving the puzzle.

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